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Solution of the relativistic bound state problem for hadrons

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Abstract

To overcome the infrared divergency problem of the Standard Model of particle physics, a second order Lagrangian – containing two boson fields – has been evaluated, which includes all mechanisms necessary to form a relativistic bound state. Strikingly, in the application to hadrons an overall consistent description of $q\bar{q}$ flavour systems is obtained only by assuming massless elementary fermions (quantons). Further, the important but not well understood confinement potential is obtained in an Abelian theory, indicating that confinement is not due to colour but is a general property of relativistic bound states.

Two mesonic systems are discussed, $\omega(782)$ and charmonium $J/\Psi(3097)$ including excited states, for which a satisfactory account of the absolute masses and a qualitative understanding of their decay widths is obtained.

PACS/ keywords: 11.15.-q, 12.40.-y, 14.40.Cs/ Bound state description of hadrons based on a second order Lagrangian with massless fermions (quantons) and two-boson coupling. Confinement and boson-exchange potential, masses and widths of $\omega(782)$ and $J/\Psi(3097)$. No coupling to scalar Higgs fields needed.

To study fundamental forces, nature provides us with hadrons and leptons, which form the constituents of matter, but also with composite systems, nuclei, atoms, and gravitational states in form of solar and galactic systems. For the description of these stable and massive systems as bound states a relativistic theory is needed, since the elementary constituents of these states are relativistic. However, relativistic bound state problems are generally difficult to solve, see e.g. Salpeter and Bethe [1], and could not be tackled so far for particle bound states (see the discussion in ref. [2]).

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Instead, powerful effective theories have been developed, which are contained (except gravitation) in the Standard Model of particle physics [3] (SM). In this framework particle bound states are included effectively by a number of parameters including nine masses of 'elementary' particles. Many experimental data are described, but the underlying mechanisms leading to particle bound states are hidden. Attempts to solve the relativistic bound state problem for hadrons in the framework of the SM were not successful [2] due to the infrared divergency of quantum chromodynamics (QCD), which makes it impossible to derive hadrons of finite radius.

So far, hadronic bound states could be simulated only in the lattice approach [4]. However, in this complex empirical approach the mechanisms which lead to bound states cannot be clearly identified. As an example, the confinement potential – of prime importance for hadron structure – is produced in lattice calculations, see e.g. the review in ref. [5], but the underlying mechanisms are still unclear, e.g. whether it can be understood in the picture of colour strings or some complicated effect of gluon binding. For a better understanding of fundamental forces it is therefore imperative to study all possible mechanisms, by which bound states of relativistic particles can be formed. This should lead to a finite theory, in which the infrared divergency problem of the SM is removed.

Different from bound state potentials in a non-relativistic theory, those of relativistic particles can be created only in complex processes, involving an interplay of fermions and bosons. By studying possible mechanisms, which could lead to such bound states, it was found that this appears possible in $q\bar{q}$ creation during the overlap of two boson fields. But this is not sufficient, it has to make sure that this system does not separate and forms a two-boson/two-fermion bound state.

To describe these processes, a second order SM Lagragian has been used, extended by two boson fields. Formally, such a Lagrangian yields solutions only, if the time component of both boson fields is the same. This allows to replace the relativistic fermion four-vectors by corresponding ones in three dimensional space, leading naturally to a non-relativistic system, for which bound state potentials can be deduced. The dynamics of such a system appears to be rather complex, but bound state properties can be studied more easily.

To facilitate such a description, instead of an extension of the non-Abelian QCD Lagrangian a simpler Lagrangian of Abelian form was used. This may to be appropriate,

since hadron systems are colour neutral. However, such a description should fail completely, if the confinement of hadrons could be generated only by the colour structure of QCD.

The Lagrangian has been used in the form

$$\mathcal{L} = \frac{1}{\tilde{m}^2} \bar{\Psi}\ i\gamma_\mu D^\mu D_\nu D^\nu \Psi \ - \ \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\ , \tag{1}$$

where $\tilde{m}$ is a mass parameter and Ψ a two-component fermion field $\Psi = (\Psi^+\ \Psi^o)$ and $\bar{\Psi} = (\Psi^-\ \bar{\Psi}^o)$ with charged and neutral part. Vector boson fields A_μ are contained in the covariant derivatives $D_\mu = \partial_\mu - igA_\mu$ and the Abelian field strength tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Only couplings to the charge ($g = g_c$) and spin ($g = g_s$) of fermions are considered[2]. In the bound state problem we deal with a finite quantum field theory, so renormalisation is not needed.

We insert $D^\mu = \partial^\mu - igA^\mu$ and $D_\nu D^\nu = \partial_\nu\partial^\nu - ig(A_\nu\partial^\nu + \partial_\nu A^\nu) - g^2 A_\nu A^\nu$ in eq. (1) and obtain for the first term of $\mathcal{L}$

$$\begin{aligned} \mathcal{L}_1 = \frac{1}{\tilde{m}^2}\bar{\Psi}\ i\gamma_\mu D^\mu D_\nu D^\nu \Psi &= \frac{i}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu \partial^\mu \partial_\nu \partial^\nu \Psi + \frac{g}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu \partial_\nu \partial^\nu \Psi \\ &+ \frac{g}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu \partial^\mu A_\nu \partial^\nu \Psi + \frac{g}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu \partial^\mu \partial_\nu A^\nu \Psi - \frac{ig^2}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu A_\nu \partial^\nu \Psi \\ &- \frac{ig^2}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu \partial_\nu A^\nu \Psi - \frac{ig^2}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu \partial^\mu A_\nu A^\nu \Psi - \frac{g^3}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu A_\nu A^\nu \Psi\ . \end{aligned} \tag{2}$$

The gauge condition $\partial_\mu A^\mu = 0$ used for simpler Lagrangians (as in QED) is replaced in our case by $\partial(\partial_\nu A^\nu) = 0$. Also for the fermion field we have to require a cancellation of the second and higher derivatives. By this condition unphysical solutions are avoided, which can arise [6] in Lagrangians, which contain higher derivatives of the fermion field.

In eq. (2) the number of field derivatives and boson couplings varies between the first and last term. This shows that the various terms are related to different kinetic situations, pointing to a rather complex dynamics of the system.

Contributions to stationary solutions have been studied by evaluating fermion matrix elements (or ground state expectation values) of the field operators $O^n_\mu(q)$ in eq. (2). These can be written by $\mathcal{M}(p'-p) = <g.s.|\ K(q)\ |g.s.> \sim \bar{\psi}(p')\ K(q)\ \psi(p)$, where $\psi(p)$ is a fermionic wave function $\psi(p) = \frac{1}{\tilde{m}^{3/2}}\Psi(p_1)\Psi(p_2)$ and $K(q = p'-p)$ a kernel, which is

[2] in the application discussed below only charge coupling is needed.

expressed by $K(q) = \frac{1}{\tilde{m}^{2(n+1)}} \; [O^n_\mu(q) g_{\mu\rho} O^n_\rho(q)]$, see e.g. [7], where n is the number of boson fields and derivatives in eq. (2) (in the present case n=3) and $g_{\mu\rho}$ the metric tensor.

In the following the essential points of the derivation of matrix elements and bound state potentials are given. For the construction of stationary states we expect contributions mainly from terms of the Lagrangian (2), which contain static fields (without derivatives). This is the case only for the last term $\mathcal{L}_{1,8} = -\frac{1}{\tilde{m}^2} \bar{\Psi} \; g^3 \gamma_\mu A^\mu A_\nu A^\nu \Psi$ and leads to a matrix element $\mathcal{M}_{3g}$, which contains three boson fields on the right and left

$$\mathcal{M}_{3g} = \frac{-\alpha^3}{\tilde{m}^8} \; \bar{\psi}(p')\gamma_\mu A^\mu(q) A_\nu(q) A^\nu(q) \; g_{\mu\rho} \; A_\sigma(q) A^\sigma(q) \gamma_\rho A^\rho(q) \psi(p) \; , \qquad (3)$$

where $\alpha = g^2/4\pi$. A comparable matrix element in the (first order) SM may be written in the form $\mathcal{M}_{SM} = \frac{-\alpha}{\tilde{m}^4} \; \bar{\psi}(p')\gamma_\mu A^\mu(q) \; g_{\mu\rho} \; \gamma_\rho A^\rho(q)\psi(p)$, giving rise to a (boson-exchange) interaction of vector structure $v_v(q) \sim \alpha A_\mu(q) A^\rho(q)$ (but only for equal times of the two boson fields, which means in the non-relativistic limit). Differently, in eq. (3) three interactions are involved, one of vector structure $v_v(q) \sim \alpha A_\mu(q) A^\sigma(q)$ or $\sim \alpha A_\nu(q) A^\rho(q)$, the other two with a coupling of two boson fields to a scalar ($\sim \alpha A_\nu(q) A^\nu(q)$ and $\sim \alpha A_\sigma(q) A^\sigma(q)$) or to a vector ($\sim \alpha A_\mu(q) A^\nu(q)$ and $\sim \alpha A_\sigma(q) A^\rho(q)$).

In a dual picture the two-boson fields, which appear twice (on the left and right side of $\mathcal{M}_{3g}$) can be regarded as interactions, but also (analoguous to the fermion wave function $\psi(p)$) as bosonic (quasi) wave functions[3] $w_s(q) = \frac{1}{\tilde{m}} A_\nu(q) A^\nu(q)$ and $w_v(q) = \frac{1}{\tilde{m}} A_\mu(q) A^\nu(q)$. However, this formal construction does not lead to a boson density for excited systems, which can be interpreted as probability. The physical picture of $\mathcal{M}_{3g}$ is that for the fundamental state of a relativistic bound state system the fermions interact only inside the two-boson density (which leads to boundary conditions discussed below) and feel therefore three interactions. A single boson-exchange interaction is possible only in dynamical situations, see the terms 2-4 in eq. (2), which do not lead to a bound state potential.

The γ-matrices as well as $g_{\mu\rho}$ can be removed (contracted) by adding a matrix element with interchanged μ and ρ (according to $\frac{1}{2}(\gamma_\mu\gamma_\rho + \gamma_\rho\gamma_\mu) = g_{\mu\rho}$). Further, an equal time requirement of the two-boson fields (to reach overlap) allows to replace all fermion four-vectors[4] by three-vectors in momentum or r-space. Correspondingly, the boson vectors

[3] leading to boson (quasi) densities $w^2(q)$ with dimension $[GeV]^2$.

[4] in a $(t, \vec{r})$ representation

are reduced to two dimensions. This yields

$$\mathcal{M}_{3g} = \frac{-\alpha^3}{\tilde{m}^5}\ \bar{\psi}(p') w_{s,v}(q)\ v_v(q)\ w_{s,v}(q)\psi(p)\ . \tag{4}$$

The wave functions $w_s(q)$ and $w_v(q)$ are given by the overlap of two boson fields with spin=0 and 1, respectively. The interaction $v_v(q)$ has a structure similar to $w_v(q)$. Writing the matrix element by $\mathcal{M}_{3g} = \bar{\psi}(p')\ V_{3g}(q)\ \psi(p)$, we obtain a three-boson potential

$$V_{3g}^{s,v}(q) = \frac{-\alpha^3}{\tilde{m}^2}\ w_{s,v}^2(q) v_v(q)\ . \tag{5}$$

Fourier transformation to r-space leads to a folding potential

$$V_{3g}^{s,v}(r) = -\frac{\alpha^3}{\tilde{m}\hbar} \int dr' w_{s,v}^2(r')\ v_v(r-r')\ . \tag{6}$$

Eq. (6) shows the typical form of a folding potential. Such a form has been used to describe elastic and inelastic hadron processes [8].

Concerning the fermion wave function $\psi(r)$ in the matrix element, here only $\psi(r) \sim w_s(r)$ is assumed. In combined with the two potentials (6), this gives rise to two vector states, a 1^- fundamental state and one of significantly higher mass, which can decay into two mesons or baryons. Other wave functions are possible, e.g. a p-wave $\psi_{L=1}(r)$ leading to 0^+ states, but these are not considered in the present paper. Since the fermionic and bosonic wave functions are finite, also the corresponding interactions are finite, in particular $v_v(r)$. Consequently, solutions of the relativistic bound state problem exist in full space, as expected for a finite theory.

To evaluate the potentials $V_{3g}^{s,v}(r)$, the boson wave functions $w_{s,v}(r)$ have to be determined. To achieve this, boundary conditions can be formulated by requiring that the interaction takes place inside the bound state volume of the g.s. As a consequence, the deduced potentials (6) should be proportional to the density $\psi^2(r) \sim w_s^2(r)$. For the potential $V_{3g}^s(r)$ this is not possible, but for the potential $V_{3g}^v(r)$ we can require

$$c_1\ w_s^2(r) \sim |V_{3g}^v(r)|\ , \tag{7}$$

if the density $w_v^2(r)$ has a radial node. Consequently, $w_v^2(r)$ must have a p-wave structure, which is translationally invariant and fulfills the condition $< r_{w_v^2} > = 0$. To achieve this, $w_v^2(r)$ is used in the form

$$w_v^2(\vec{r}) = w_v^2(r)\ Y_{1,m}(\theta,\Phi) = w_{v_o}^2 (1 + \beta R\ d/dr) w_s^2(r)\ Y_{1,m}(\theta,\Phi)\ , \tag{8}$$

where $w_{v_o}^2$ is determined from the normalisation $\int rdr\ w_v^2(r) = \hbar^2$ and βR from the condition $< r_{w_v^2} > = 0$.

Using the above constraints (7) and (8) this leads to a self-consistent boson wave function of the form

$$w_s(r) = w_{s_o}\ exp\{-(r/b)^\kappa\}\ , \tag{9}$$

where w_{s_o} is determined from the normalisation $4\pi \int rdr\ w_s^2(r) = \hbar^2$ and κ from condition (7). The extra interaction $v_v(r) \sim w_s(r)$ is derived also from an expression similar to eq. (8), but here a translational invariance condition $< r_v > = 0$ does not exist. Instead, βR is adjusted to fit the boundary condition (7) at large radii.

To generate a stable bound state, the potentials $V_{3g}^{s,v}(r)$ are not sufficient to keep the two boson fields confined. Inspecting the other terms of the Lagrangian (2), these show kinematic situations, in which bosons and/or fermions are in motion. Nevertheless, term 6, which may be written in the form $\mathcal{L}_{1,6} = -\frac{ig^2}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu(\partial_\nu A^\nu)\Psi - \frac{ig^2}{\tilde{m}^2}\bar{\Psi}\ \gamma_\mu A^\mu A_\nu\ \partial^\nu\Psi$, gives rise to another bound state potential.

The first term of $\mathcal{L}_{1,6}$ leads a matrix element $\mathcal{M}_{2g}$, which contains two boson fields and a derivative on the right and left

$$\mathcal{M}_{2g} = \frac{\alpha^2}{\tilde{m}^8}\bar{\psi}(p')\gamma_\mu A^\mu(q)(\partial_\nu A^\nu(q))g_{\mu\rho}\gamma_\rho A^\rho(q)(\partial_\sigma A^\sigma(q))\psi(p)\ . \tag{10}$$

With the boson gauge condition discussed above, we can write $(\partial_\nu A^\nu(q))(\partial_\sigma A^\sigma(q)) = \frac{1}{2}\partial_\nu[\partial_\sigma(A_\mu A^\mu)^\sigma]^\nu$. After contracting the γ-matrices and reducing the fermion and boson vectors by one dimension as discussed for $\mathcal{M}_{3g}$, this yields

$$\mathcal{M}_{2g} = \frac{\alpha^2}{2\tilde{m}^6}\bar{\psi}'(p')\ w_s'(q)\partial^2 w_s'(q)\ \psi'(p)\ , \tag{11}$$

where $\psi'(q)$ and $w_s'(q)$ represent fermion and boson wave functions with a q-dependence different from $\psi(q)$ and $w_s(q)$ in $\mathcal{M}_{3g}$. Since the two bosons are bound (momentarily), see eq. (4), they have a kinetic energy $\partial^2 w_s'(q)/2\tilde{m}$. According to the virial theorem this implies also the existence of a static two-boson potential $V_{2g}(q)$.

In a transformation to r-space the bosonic part of eq. (11) gives rise to a Hamiltonian of the form

$$-\frac{\alpha^2\tilde{m} < r^2_{w_s'^2} >}{2}\ \Big(\frac{d^2 w_s'(r)}{dr^2} + \frac{2}{r}\frac{dw_s'(r)}{dr}\Big) + V_{2g}(r)\ w_s'(r) = E_i\ w_s'(r)\ , \tag{12}$$

where $w'_s(r)$ is the Fourier transform of $w'_s(q)$ given by $w'_s(r) = \frac{2\pi}{\hbar\tilde{m}^2} \int qdq\ j_o(qr)\ w'_s(q)$ and $< r^2_{w'^2_s} >$ is the radius square of the boson density. The potential $V_{2g}(r)$ is given by

$$V_{2g}(r) = \frac{\alpha^2 \tilde{m} < r^2_{w'^2_s} >}{2} \Big(\frac{d^2 w'_s(r)}{dr^2} + \frac{2}{r}\frac{dw'_s(r)}{dr}\Big)\frac{1}{w'_s(r)} + E_o\ , \tag{13}$$

where E_o is the energy of the lowest eigenstate.

A similar potential involving $w'_v(q)$ deduced from $\mathcal{L}_{1,7}$ yields negligible contribution to the binding energy due to strong cancellations. All other terms of the Lagrangian (2) do not contribute to bound state potentials.

Including the potential $V_{2g}(r)$ a final boundary condition for the matching of density and potentials can be formulated, requiring that the sum of all three potentials $V^{s,v}_{3g}(r)$ and $V_{2g}(r)$ match the densities $\psi(r) \sim w^2_s(r)$. However, $V_{2g}(r)$ cannot be added directly, since it is derived from a kinetic energy term, which itself is generated from the two-boson field $w'_s(r)$. Therefore, an adequate boundary condition can be formulated by

$$c_2\ w^2_s(r) = |V^s_{3g}(r)| + |V^v_{3g}(r)| + V'_{2g}(r)\ , \tag{14}$$

where $V'_{2g}(r) \sim (\alpha^2/2\tilde{m}) w'^2_s(r)$ is proportional to the density $w'^2_s(r)$, given by the same radial form in eq. (9) but with a smaller slope parameter b'. This relation underlines the close ties between the different potentials.

To facilitate the calculation of the binding energies, separate bound state solutions were computed for $V_{3g}(r)$ and $V_{2g}(r)$ using the fermionic wave functions $\psi_{3g}(r) \sim w_s(r)$ and $\psi_{2g}(r) \sim w'_s(r)$. Further, bound state energies for excited states in $V_{2g}(r)$ were deduced by fitting modified Airy functions [9] to the potential. This appears to be a reasonable approximation, confirmed by the overall self-consistency achieved.

The mass of the system is defined by the energy to balance binding

$$M^{s,v}_n = -E^{s,v}_{3g} + E^{\ n}_{2g} + 2m_q\ , \tag{15}$$

where $E^{s,v}_{3g}$ is the negative binding energy in $V^s_{3g}(r)$ and $V^v_{3g}(r)$ (for these potentials only the lowest state is discussed here), $E^{\ n}_{2g}$ positive binding energies in $V_{2g}(r)$, and $2m_q$ the mass of the elementary fermions (as assumed in the quark model). This shows two types of mass generation, binding in the Coulomb like potential $V_{3g}(r)$ and dynamical mass

generation in $V_{2g}(r)$. The lowest binding energies E^s_{3g} and E^1_{2g} can be related to the average momentum of the bosonic density $< q >= \int q dq \; w^2_s(q) / \int dq \; w^2_s(q)$ by the energy-momentum relation, which yields $E^1_{2g} =< q > + E^s_{3g} + 2m_q$. From this, a further boundary condition is obtained

$$M_{g.s.} = < q > + 2m_q \; . \tag{16}$$

Now, all conditions are specified. However, by calculating relativistic bound states for different flavour systems with fermion masses m_q taken from the quark model, a consistent solution could be obtained only for light $q\bar{q}$ systems, as $\omega(782)$, for which m_q is assumed to be very small. In case of c- or b-quarks, a satisfactory solution could not be found.

However, an overall internally and externally consistent solution for all systems studied is obtained by assuming massless elementary fermions (quantons). The internal consistency refers to the mass parameter $\tilde{m}$, which is in this case the reduced mass due to the binding energies in $V_{2g}(r)$ and $V_{3g}(r)$. For the matrix element $\mathcal{M}_{3g}$ (containing two bosons and two fermions) the reduced mass m is given by $m = \frac{1}{4}|E^s_{3g}|$, whereas for $\mathcal{M}_{2g}$ $m = \frac{1}{2} E_{2g}$. Therefore, the reduced mass $\tilde{m}$ should be between the limits

$$\frac{1}{2} \; E_{2g} \geq \tilde{m} \geq \frac{1}{4} \; |E^s_{3g}| \quad or \quad \frac{1}{2} \; E_{2g} < \tilde{m} < \frac{1}{4} \; |E^s_{3g}| \; . \tag{17}$$

This relation is used as additional constraint.

The external consistency concerns the description of different flavour systems, which in case of massless fermions cannot be distinguished by the fermion masses m_q. However, in our formalism different flavour systems are described by different slope parameters b in eq. (9), which are related by a vacuum potential sum rule [10]. In this way all flavour systems are connected.

The need for massless fermions has important implications. First, the vacuum of the applied theory is the absolute vacuum with average energy $E_{vac} = 0$. This is consistent with the low energy density of the universe deduced from astrophysical observations. Second, in the potential $V_{2g}(r)$ the lowest energy solution is the vacuum and therefore $E_o = 0$. By this, the absolute height of $V_{2g}(r)$ is fixed. Third, by rewriting eq. (11) in the form $\mathcal{M}_{2g} = \frac{\alpha^2}{2\tilde{m}^6} w'_s(q) \{\bar{\psi}(p')\psi(p)\} \partial^2 w'_s(q)$, one can see that fermion-antifermion pairs can be created during the overlap of two fluctuating boson fields. By this mechanism stable particles can be created out of the absolute vacuum.

In the whole formalism there are finally five constraints (7), (14), (15), (16) and (17), by which **all** five open parameters, two slope (or size) parameters b and b', a shape parameter κ, the mass parameter $\tilde{m}$ and the coupling constant α, are determined within certain ambiguities. In addition, the relation of the slope parameter b of different flavour states by a vacuum sum rule [10] indicates that in principle a complete solution of the relativistic bound state problem for all flavour states is achieved.

An evaluation of the above formalism is discussed for two mesonic systems, $\omega(782)$ and charmonium $J/\Psi(3097)$ including excited states. The potentials $V_{3g}(r)$ and $V_{2g}(r)$ have been determined by adjusting the open parameters to the five constraints discussed above. Remaining uncertainties have been reduced by adjusting the confinement potential $V_{2g}(r)$ to fit the spectrum of radial excitations, which allows a rather precise determination of the mass parameter $\tilde{m}$. Nevertheless, some ambiguities remain, mainly between κ and α.

Results on the radial dependence of densities and potentials are given in figs. 1 and 2 by using the parameters in table 1. In the upper part the interaction $v_v(r)$ is given by the solid line. Compared to the Coulomb potential $v_{coul}(r) = \hbar/r$ (dot-dashed line) there are no divergencies for $r \to 0$ and ∞, in agreement with the demand of a finite theory.

In the middle part a comparison of the density $w_s^2(r)$ (dot-dashed line) with the potentials $V_{3g}^s(r)$ (dashed line) and $V_{3g}^v(r)$ (solid line) is made. We see that condition (7) for the vector potential is well fulfilled at larger radii. This indicates that the bosonic wave function $w_s(r)$ is well described by the radial form in eq. (9) and that also relation (8) between $w_s(r)$ and $w_v(r)$ is correct.

In the lower part the scalar two-boson density $w_s^2(r)$ (upper dot-dashed line) is compared to the sum of all potentials. Only a considerably smaller slope parameter b' meets the boundary condition (14), showing a good consistency of our description. The resulting masses, momenta and mean radius squares are given in table 1 together with the final parameters. For the determination of α, the angular momentum and spin coupling coefficients $< q\bar{q}\ |L|\ 1^- >$ (for L=0 and 2) are taken into account as well as the corresponding components $v_v^L(r)$ of the multipole-expansion of the interaction $v_v(r) = \sum v_v^L(r)$, given in the present case by $v_v^L(r) =< q\bar{q}\ |L|\ 1^- >^2 v_v(r)$.

The results for $\omega(782)$ are given in fig. 1, those for charmonium in fig. 2. Although the

Table 1: Results for two mesonic systems, $\omega(782)$ and charmonium J/ψ including excited states, in comparison with the data [3]. Masses are given (in GeV), b and b' in fm, the mean radius squares in fm^2. For $\omega(782)$ E_{2g}=0.51 GeV and E^s_{3g}=-0.27 GeV, for $J/\psi(3097)$ E_{2g}=0.34 GeV and E^s_{3g}=-2.76 GeV.

System	M_1^s	M_2^s	M_3^s	M_4^s	M_1^v	M_1^{exp}	M_2^{exp}	M_3^{exp}
$\omega(782)$	0.78	1.42	1.93		3.2	0.782	1.42±0.03	
J/ψ	3.10	3.69	4.16	4.58	12.7	3.097	3.686	4.16±0.02

System	b	b'	κ	$\tilde{m}$	α	α^*_{eff}	$< r^2_{w_s} >$	$< r^2_{w'_s} >$
$\omega(782)$	0.465	0.220	1.42	0.25	1.05	0.14	0.16	0.04
J/ψ	0.097	0.050	1.39	0.26	0.83	0.10	0.71 10^{-2}	0.19 10^{-2}

* $\alpha_{eff} = \alpha \int dr \ v_v(r) / \int dr \ v_{coul}(r)$

binding energies are quite different, in both cases a satisfactory agreement of the various quantities is obtained, which fulfil all boundary conditions. It is important to mention that in a transformation to momentum space agreement of density and potential is obtained only for massless elementary fermions.

Apart from the requirement of massless fermions, surprising results are obtained also for the potential $V_{2g}(r)$, which is shown in fig. 3. This has the same form as the known 'confinement' potential $V_{conf} = -\alpha/r + l \cdot r$ deduced from potential models [11]. Also the comparison with the 'confinement' potential from lattice simulations of Bali et al. [12] (solid points with error bars) shows a very similar behaviour. This indicates clearly that the confinement of hadrons is a general property of relativistic bound states and can be generated in an Abelian or non-Abelian theory. The question of a vector or scalar structure of the confinement potential can be studied by looking at the splitting of p-wave states in charmonium and bottonium, see ref. [13]. From the existing data neither a vector nor a scalar structure is found, supporting the derivative structure of the potential $V_{2g}(r)$.

As a last point a brief discussion of mass distributions is made (related to the widths and decay probabilities), which goes beyond a static description of bound state properties. Experimentally, the widths of the mesonic states considered are quite different, 8.4 MeV for the first and 90 keV for the second one [3]. The small width of J/ψ is explained in

the quark model by a different quark structure involving c-quarks, which cannot decay into light quarks. In the present model the width is related to the dynamical structure of the system, given by two kinetic energy distributions $T_{2g}(q)$ and $T_{3g}(q)$. Their Fourier transforms $T_{2g}(r)$ and $T_{3g}(r)$ are related to the potentials $V_{2g}(r)$ and $V_{3g}(r)$ by the virial theorem $T_{2g}(r) = V_{2g}(r)$ and $T_{3g}(r) = \frac{1}{2} < r^2 > (d^2V_{3g}(r)/dr^2 + \frac{2}{r}\, dV_{3g}(r)/dr)$. From $T_{2g}(q)$ and $T_{3g}(q)$ the decay probability of state 1 to state 2 is given by $p^{2g}_{(1\rightarrow 2)} = [T^1_{2g}(q)\; T^2_{2g}(q)]^{1/2}$ and $p^{3g}_{(1\rightarrow 2)} = [T^1_{3g}(q)\; T^2_{3g}(q)]^{1/2} \int d\Omega\; Y_{l_1}(\theta,\phi) Y_{l_2}(\theta,\phi)$ together with mixed terms.

Results are given in fig. 4, which show that the overall width of the mass distribution is dominated by the two-boson kinetic energy $T_{2g}(q)$ (related to the confinement potential). In the Fourier transformation[5] this yields a sharp peak with a width of about 0.4 and 0.5 GeV for the low and high mass system, respectively (solid lines). Differently, $T_{3g}(q)$ yields a wide mass distribution, which (including threshold effects in the form used in ref. [14]) is given by dot-dashed lines.

In both cases the experimental width of the fundamental state is much smaller than obtained in our calculations. This can be understood by a rather stable 1^- g.s. with a strongly delayed decay. Indeed, $\omega(782)$ decays almost exclusively to three pions, the decay to two pions is strongly suppressed.

In the case of J/ψ the experimental width of the g.s. is four orders of magnitude smaller ($\sim$ 90 keV) than the narrow peak in the calculation, indicating a system with a strongly delayed decay. The decay probability $p_{(J/\psi\rightarrow\omega(782))}$ is less then 10^{-10} for $T_{2g}(q)$ and extremely small for $T_{3g}(q)$. This is mainly due to the large width of $T_{3g}(q)$, which leads only to a tiny decay fraction of $4 \cdot 10^{-4}$ from the region of the 90 keV wide J/ψ peak. Qualitatively this yields decay widths in agreement with the data, but more detailed work is needed to calculate specific decay channels with good precision. Here, only one decay should be mentioned, $J/\psi \rightarrow e^+e^-$ with a decay width of 5.16 keV. By taking the results for the electron from ref. [15] and applying a folding of two electron distributions, a decay probability for this channel of $0.8 \cdot 10^{-5}$ is obtained, corresponding to a width of 4.8 keV, which is close to the experimental value.

[5] for convergence of the long range potential $V_{2g}(r)$, a Gaussian smoothing is applied.

In summary, a solution of the relativistic bound state problem for hadrons has been derived from a second order Lagrangian, which may be considered as the most natural way to overcome the infrared divercency of the SM. The application to $q\bar{q}$ mesons has led to surprising results, a consistent description of different flavour systems by assuming massless elementary fermions (quantons). This has the consequence that the Higgs-mechanism as well as supersymmetric extensions of the SM (in which the infrared divergency problem has not been removed) are not needed. Further, the known confinement potential is well understood in an Abelian theory (without colour degree of freedom). Our solution is self-consistent, without free parameters, and shows a relation to the absolute vacuum. It may therefore correspond to a rather fundamental description of hadrons with a simpler symmetry structure (less degrees of freedom) than the present (first order) form of the SM.

In a further paper [16] an application of the present formalism to heavy $q\bar{q}$ bound states in the "top"-mass region is discussed with results on the structure of Z(91.2 GeV) and the new scalar state at 126 GeV, discovered recently in Atlas and CMS data (tentatively interpreted as Higgs-boson).

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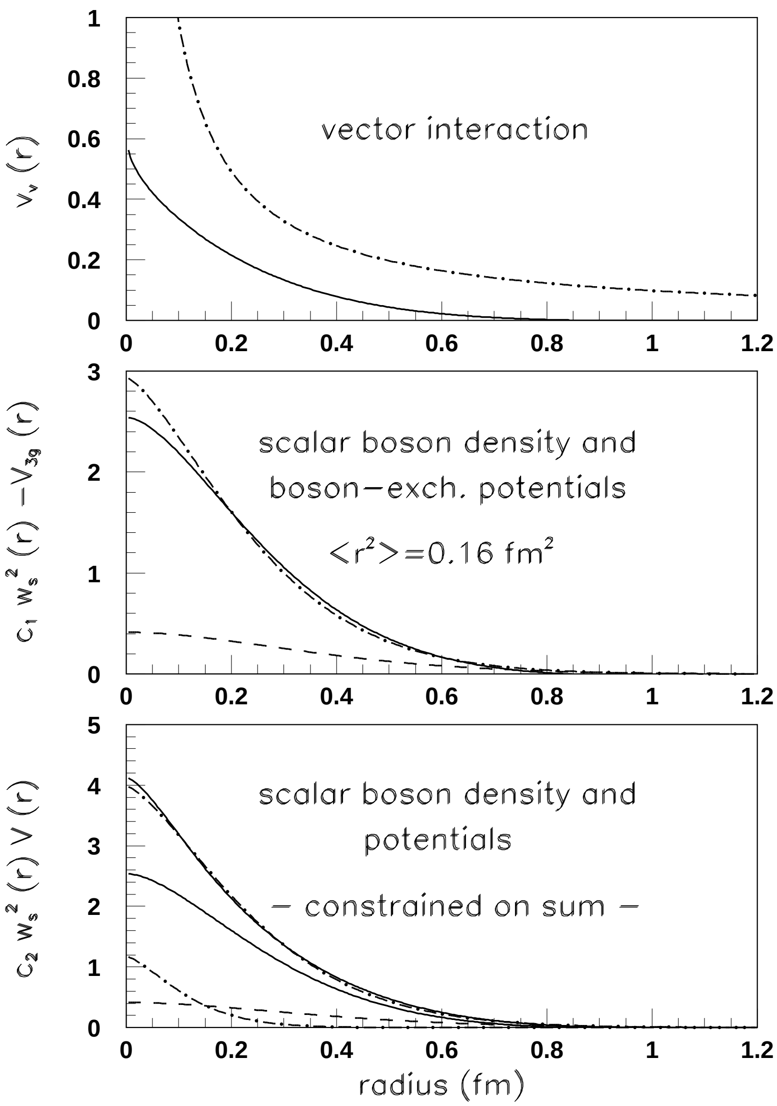

Figure 1: Self-consistent solution for a mesonic system corresponding to $\omega(782)$. Upper part: Interaction $w_v(r)$ in comparison with the Coulomb potential (for α=0.3), given by solid and dot-dashed lines, respectively. Middle part: Bosonic density $w_s^2(r)$ and potential $|V_{3g}^v(r)|$ given by the overlapping dot-dashed and solid lines, respectively, matched by the condition (7); $|V_{3g}^s(r)|$ is shown by dashed line. Lower part: Potentials $V_{2g}'(r)$ (lower dot-dashed line), $|V_{3g}^s(r)|$ (dashed line), $|V_{3g}^v(r)|$ (lower solid line) and sum (solid line) in comparison with $w_s^2(r)$ (upper dot-dashed line) to meet condition (14).

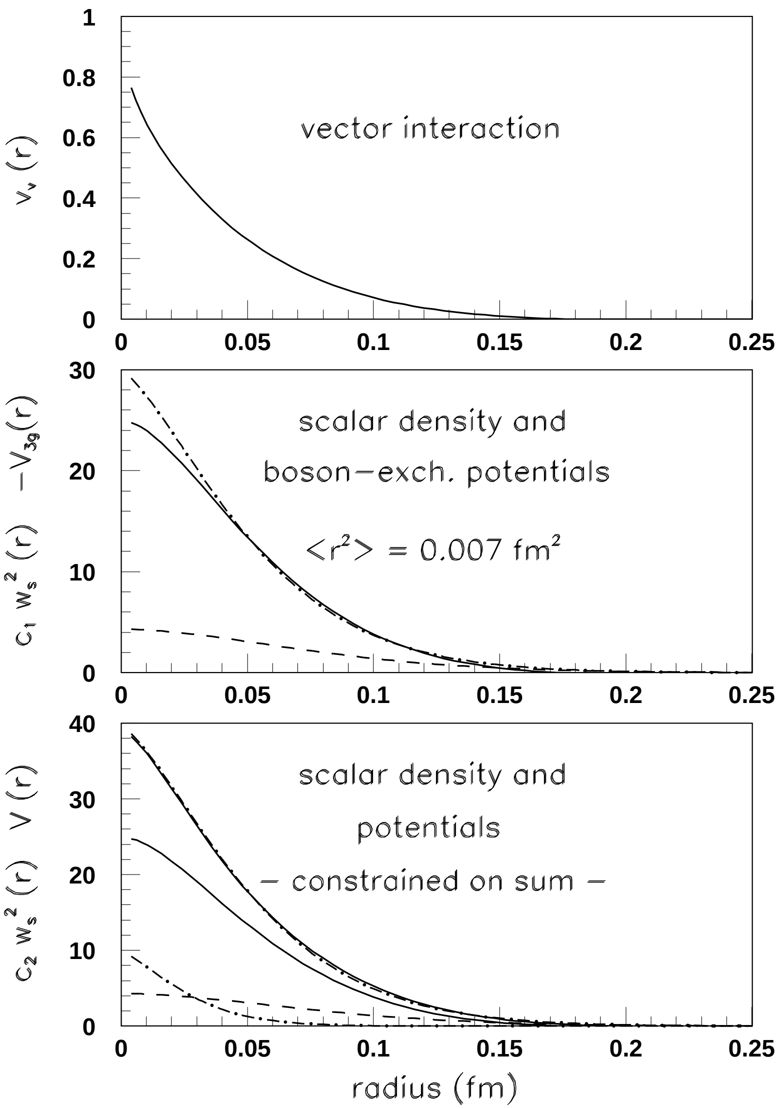

Figure 2: Same as figure 1 for the charmonium system $J/\psi(3097)$.

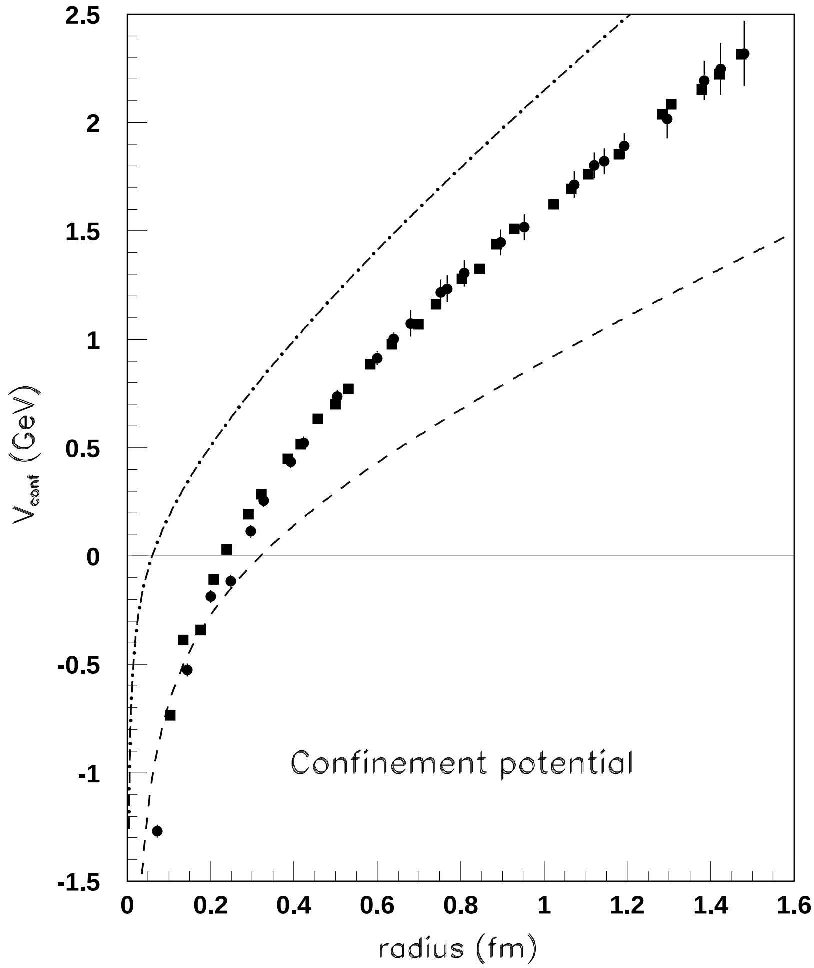

Figure 3: $V_{2g}(r)$ for the two mesonic systems $\omega(782)$ and charmonium J/ψ, given by dashed and dot-dashed line, respectively, and comparison with the confinement potential from lattice gauge calculations [12] (solid points).

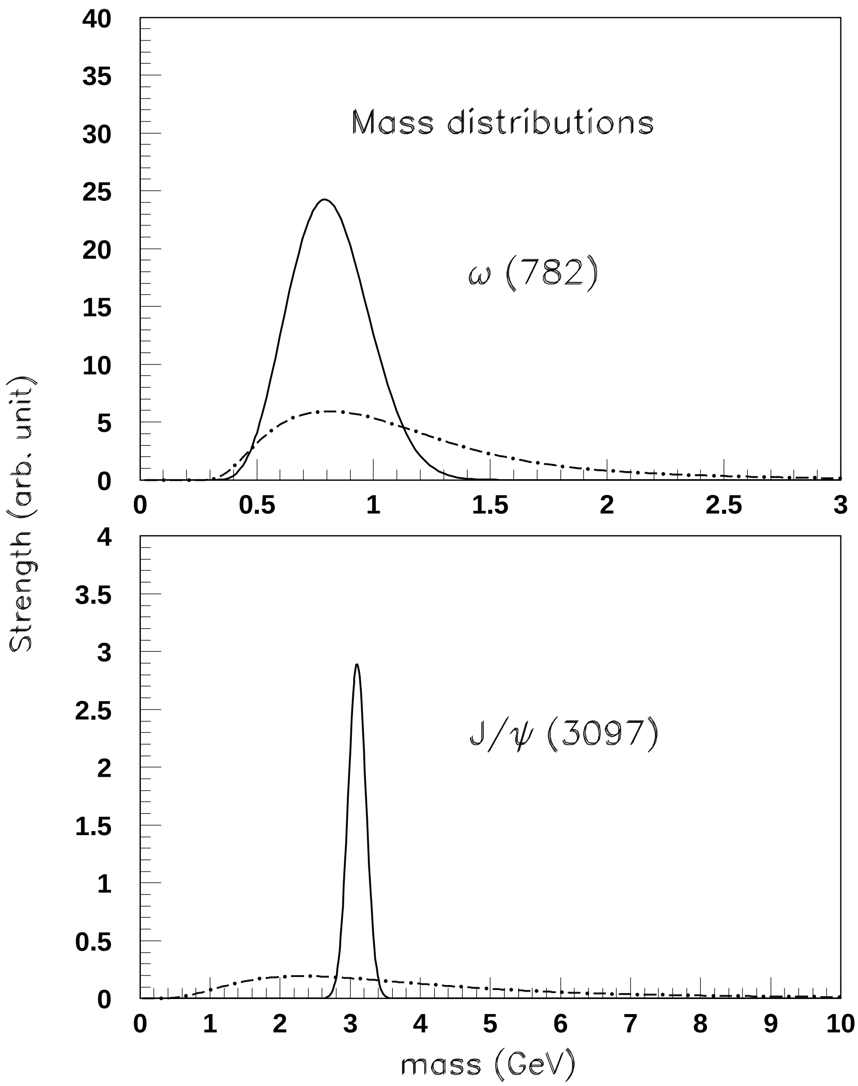

Figure 4: Mass distributions for the two systems. The dominant contribution is due to the two-boson kinetic energy term $T_{2g}(q)$ (narrow solid lines), whereas $T_{3g}(q)$ is given by wide dot-dashed distributions (enhanced by a factor 10). The widths are much larger than found experimentally, indicating a strongly reduced decay probability.